Black Holes for Bedtime

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"Thinukidanu Phaniraya Ramayanada bharadali".
"The King of Snakes groaned under the weight of the Ramayanas"
—From the Mahabharatha by Kumara Vyasa.*

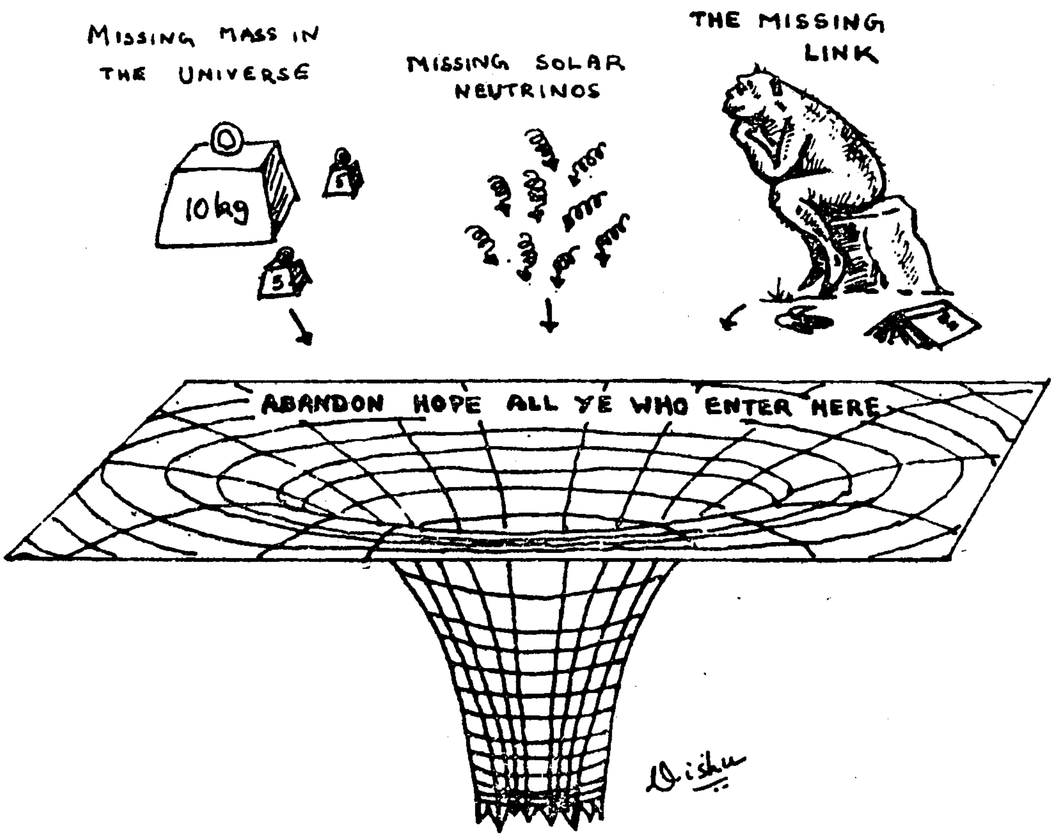
In the year 1756 Siraj-ud-daula, the Nawab of Bengal tried to create a black hole by squeezing a large amount of matter, in the readily available human form, into a rather small volume. The experiment was a failure and the political consequences were disastrous. But the name black hole came into existence. Only in the last few years have we come to realize that the Nawab's method of manufacturing black holes is feasible after all, but the compressions required are of cosmological proportions. These are the primordial black holes which may have nothing to do with politics.

About two hundred years later, the term black hole was introduced into astrophysics. Since then it has come to represent many things in popular imagination: symbol of Dantesque doom, panacea for astrophysical ills, inexhaustible source of energy, free ticket to time travel, gateway to other universes which may not exist, explanation for the missing mass in the universe, the missing solar neutrinos, the missing link—in general anything that is missing. Fortunately, behind all the folklore and fantasy, fallacy and fiction, stands the black hole of the sane scientist, which is a product of Einstein's general theory of relativity and a manifestation of high space-time curvature.

The first general relativistic black hole—the Schwarzschild black hole—was born immediately after the advent of the theory itself. But it lay dormant for nearly half a century collecting dust in old books and obscure articles. This was ironical in view of the fact that as early as in the thirties mass limits for stable stellar configurations had been derived and the dynamics of spherical collapse leading to the formation of a black hole had been amply demonstrated. Nevertheless, in the past decade, intense research—perhaps unprecedented in the history of relativity—has clarified

*The reason for his choosing the Mahabharatha rather than the Ramayana for rendering in the Kannada language, says the poet Kumara Vyasa, is that the various versions of the latter have already burdened heavily the seven-hooded Adisesha, the King of Snakes, who supports the earth. Perhaps so have the articles on black holes. The present review is then a minor perturbation, an admissible one it is hoped, on the already existing mass of literature.

Gravitation, Quanta and the Universe: Proceedings of the Einstein Centenary Symposium held at Ahmedabad in 1979, Ed. A R Prasanna et al. Wiley Eastern 1980



the properties of black holes and their relevance to astrophysics to a great extent. The subject is vast and the space available here limited. We shall not attempt therefore to paint the portrait of a black hole with "pimples, warts, and all"—especially since the black hole, as we shall see, cannot have them. Only a thumbnail sketch will be aimed at. Let us then begin at the beginning.

In the beginning, Einstein created the space-time and the line element. And the space-time was flat and empty. Einstein introduced matter and gravitation. The space-time became curved. And the line element was given by the metric tensor which had ten components in general. These ten functions of space and time also incorporated relevant parameters like the mass and angular momentum of the source of gravitation. And they told the entire gravitational tale of the particular situation on hand.

The Schwarzschild empty space line element, which incorporates the black hole, has the form:

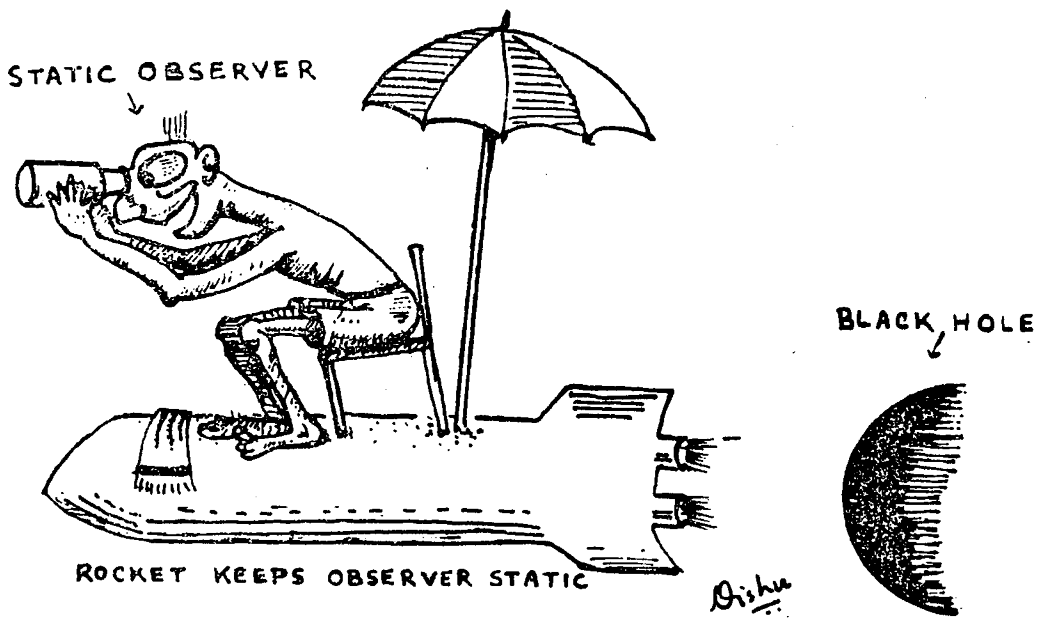
$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \left(1 - \frac{2m}{r}\right)^{-1} dr^2 - r^2 (d\theta^2 + \sin^2\theta \, d\phi^2).$$

This is valid outside a spherically symmetric source of mass m ($G = C = 1$ as usual, so that mass is expressed in units of length). Just with the introduction of the Newtonian potential m/r into the metric, a change that seems trivial, we leave the safe moorings of a flat space-time and enter the field of a black hole. The spherical surface $r = r_g = 2m$ represents the black hole.* It used to be called 'the Schwarzschild singularity', since the metric becomes pathological there, but we know that this is only due to the wrong choice of co-ordinates. For a long time, this surface was at best ignored and at worst regarded with annoyance, but was never taken seriously. The reason is that the Schwarzschild—or the gravitational radius r_g has an extremely small value compared to the size of ordinary objects in nature. For example, r_g has the value of 3 kilometers for the sun and 10^{-23} cms in the case of a man weighing 143 pounds. This distance lies well within the mass distribution where the metric is completely different and the concept of a black hole does not even exist. There is no black hole deep within a man's heart and a knife plunged in does not form an accretion disk. The object must become so compact that the surface r_g, in order to be a black hole, has to be outside in the empty space. There is a story of a man on a crash diet whose weight decreased drastically while his volume remained unaltered; the man started to float in the air. Had his weight increased instead, the man could have turned into a black hole. Mercifully this does not happen in nature. On the other hand, a sufficiently massive star is expected to collapse continuously after exhausting its fuel and form a black hole. This process can be studied exactly if the star is spherical in shape. Outside the star, the space-time is the Schwarzschild exterior and as the star shrinks, it leaves behind in its wake more and more of this exterior. The mass collapses into the Schwarzschild radius thereby liberating the black hole and finally hits the singularity at the origin.

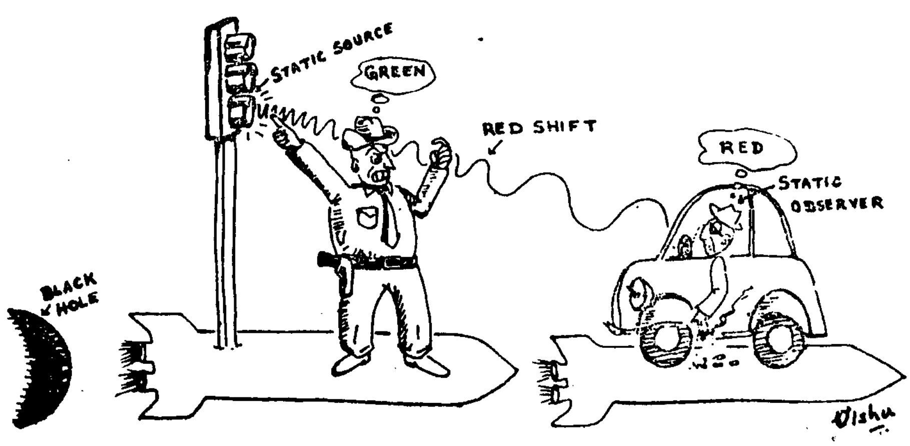
What exactly is a black hole? A systematic study of this question in regard to both the Schwarzschild and the Kerr black holes was undertaken in the late sixties. We start the discussion by noting some very basic properties of the Schwarzschild metric. The metric components are independent of time. Furthermore, there are no terms like $dt\, d\phi$ in the metric. The latter observation corresponds to the fact that the source is not rotating and consequently no rotation is built into the space-time either. The time-independence of the metric shows the existence of a very special vector field which spells out the inherent symmetry of the space-time. This is known as the Killing vector which in the usual Schwarzschild coordinates has the form $\xi^a = (1, 0, 0, 0)$, namely the direction of time. Motion along the vector is tantamount to the flow of time which brings no change in the space-time picture. We can define sources and observers with four-velocities along the trajectories of the Killing vector at each spatial point. These will then be the static observers who do not

*There seems to be some confusion of terminology in the literature. Sometimes the entire spacetime region containing the surface $r = 2m$ as well as the singularity at $r = 0$ is referred to as the black hole. On the other hand terms like the "area of a black hole" imply reference only to a surface. In this article the black hole is consistently identified with the Schwarzschild surface $r = 2m$ or its equivalent in any other spacetime.

move; only time passes on as they sit around and watch. However, the Killing vector is timelike only outside the black hole. It is lightlike inside. Consequently, the static observers are defined only up to the black hole. With no reference to any particular coordinate system, we can characterise the Schwarzschild black whole as the surface on which the timelike Killing vector becomes null making it thereby the so-called 'static limit'.



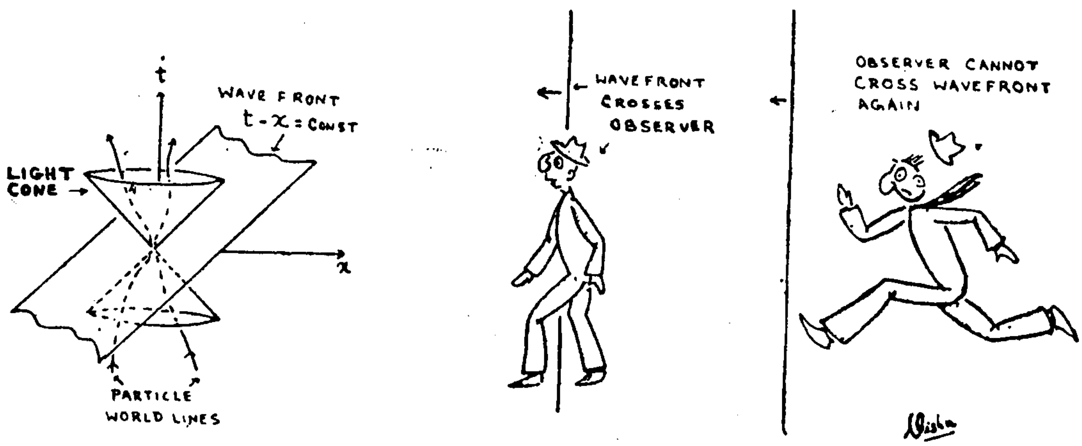
An immediate consequence of the above feature pertains to the gravitational redshift with respect to static sources and observers. As in any other branch of physics, the existence of time symmetry leads to a conserved quantity. Along a geodesic, which represents the natural particle motion inherent to a given space-time, the scalar $\xi_a p^a$ is constant. Here p^a is the four-momentum tangential to the geodesic. At the same time, an observer with a four-velocity u^a measures the energy E of the geodesic particle as $u_a p^a$. Applying these two considerations to a photon travelling along a null geodesic from a static source to a static observer, one readily obtains the ratio of the observed and source frequencies as



$$\frac{\nu_0}{\nu_s} = \frac{(\xi_a \xi^a)_s^{1/2}}{(\xi_a \xi^a)_0^{1/2}} = \frac{(1 - 2m/r_s)^{1/2}}{(1 - 2m/r_0)^{1/2}}\,.$$

As the source approaches the black hole, the observed frequency tends to zero or the gravitational redshift becomes infinite. The Schwarzschild black hole is an infinite redshift surface for static sources.

We now come to the definitive property of the black hole. It is a null surface and hence acts as a one-way membrane. That is, if the length of the normal to the surface is computed using the metric tensor, the length turns out to be zero. Null surfaces are quite familiar to us in everyday life even in flat space-time. Every wavefront is a null surface. Such a surface is tangential to the local light cone. Timelike trajectories which

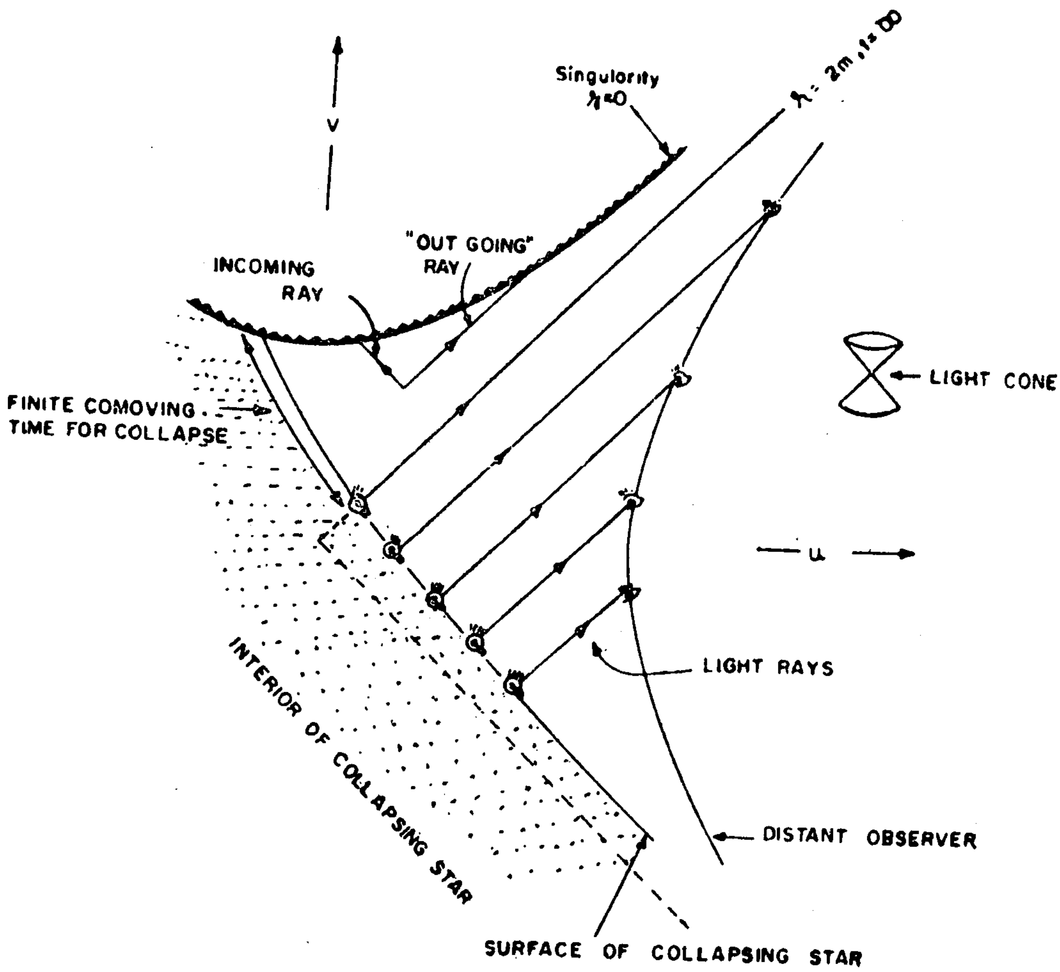


have to be confined to within the coue necessarily cross the wavefront in one direction and cannot cross it again in the reverse direction. In other words, once the wavefront has crossed a material particle, the latter cannot overtake the front and cross it the other way round. Here the wavefront travels in space with the elapse of time. And it is at spatial infinity at some time or the other. The black hole, being a null surface, is similar to the wavefront although it is devoid of any physical content such as the electromagnetism associated with the wavefront. The light cone is again tangential to it everywhere. A particle falling in, having crossed it in one direction, cannot turn back and recross it in the opposite sense. It acts thus as a one-way membrane. However, unlike the wavefront, it does not 'travel' in time and does not extend to spatial infinity. This is a reflection of the gravitational pull exerted by the mass that has fallen in, thereby creating the black hole. To summarise, the Schwarzchild black hole happens to be the surface on which the timelike Killing vector has become null and is hence the static limit; furthermore, it is a null surface and consequently behaves like a static one-way membrane. Since not even photons, let alone material particles, can escape from within this surface, no event occurring in this region can communicate itself with the outside world. The surface is therefore referred to as an 'event horizon'.

The true picture of the black hole, free of co-ordinate pathologies, emerged with the discovery of the Kruskal co-ordinates. The radial co-ordinate r and the time t are combined in a somewhat complicated manner to produce the new co-ordinates u and v in terms of which the line element reads

$$ds^2 = f^2(dv^2 - du^2) - r^2(d\theta^2 + \sin^2\theta d\phi^2),$$

where $f^2 = (32m^3/r) \exp(-r/2m)$. The metric is now regular all the way down to the singularity at the origin. Everywhere the light cones are similar to those of the flat space-time. It can be readily seen that the light cone is tangential to the black hole. Inside, both incoming and 'outgoing' rays hit the singularity, so much so a particle that has entered the black hole is fated to do the same and get crushed. The picture of stellar



KRUSKAL DIAGRAM

collapse is also clear. Intervals of light pulses emitted from the surface are progressively dilated as seen by an outside observer whose trajectory is a hyperbola in the u-v plane. The surface of the star seems to take infinite amount of the observer's time to reach the Schwarzschild radius. A similar effect is that the colour of the light reaching the observer shifts more and more to red and the intensity dies down exponentially. Old stars never die; they just fade away. The proper infall time of the surface, however, is finite for the entire collapse. In addition to displaying all this in a straightforward manner, the Kruskal diagram becomes the proper framework in which many phenomena are studied, as the metric no longer suffers from the spurious divergences at the Schwarzschild radius.

Orbits around the black hole and their stability have been worked out in detail. These computations are of significance to astrophysics. Another effect of some importance is the tidal distortion of extended objects in the vicinity of a black hole. Gruesome descriptions have been given of this effect on a man falling into a black hole. Since at the Schwarzschild surface the tidal force is proportional to $1/m^2$, one is advised to fall into a massive black hole, if fall one must, in order to enjoy the trip before gravity takes its final toll. This would happen as the singularity at the origin is approached where the tidal forces become infinite.

So far we have been concentrating on a static black hole. Let us now turn to the rotating black hole of the Kerr metric:

$$ds^2 = \left(1 - \frac{2mr}{\Sigma}\right) dt^2 - \frac{4mar}{\Sigma} \sin^2\theta \, dt \, d\phi - \frac{\Sigma}{\Delta} dr^2$$

$$- \Sigma \, d\theta^2 - \left(r^2 + a^2 + \frac{2ma^2 r \sin^2\theta}{\Sigma}\right) \sin^2\theta \, d\phi^2$$

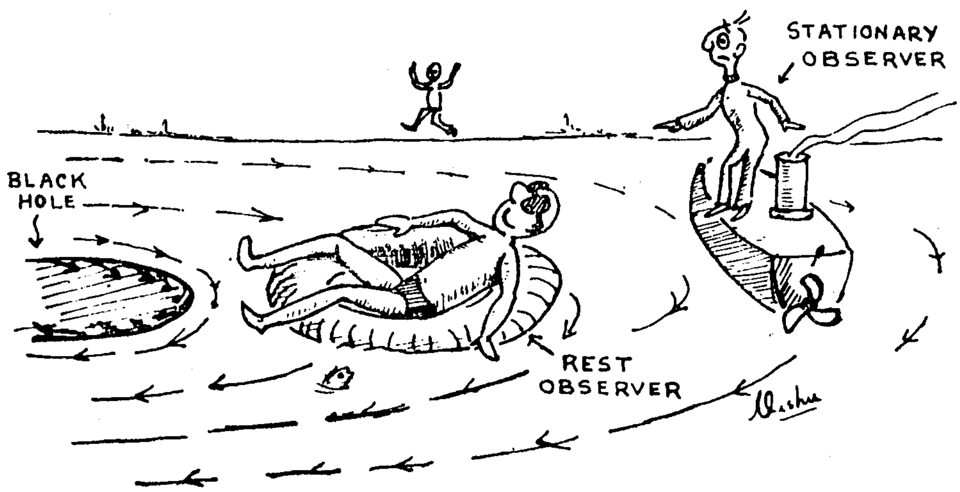
with $\quad \Delta = r^2 - 2mr + a^2$ and $\Sigma = r^2 + a^2 \cos^2\theta,$

where m is the mass of the black hole and a is its angular momentum per unit mass. You do not have to scrutinize the terms in the metric carefully and endanger your eyesight. It suffices to note that there is a $dt\,d\phi$ term which represents the rotation imparted to the space-time by the black hole, while the metric components are still independent of time. Such a space-time is said to be 'stationary'. Further, when the parameter a is set equal to zero, we recover the Schwarzschild exterior. In order to analyze the Kerr space-time, we appeal once again to our old friend the Killing vector which has the same form as before. It is timelike sufficiently far away from the black hole, but becomes null on the surface

$$r = m + (m^2 - a^2 \cos^2\theta)^{1/2},$$

which is obtained by setting the coefficient of dt^2 equal to zero. Stationary observers following Killing trajectories can be defined only outside this surface and hence it is the 'stationary limit'. It is consequently the infinite redshift surface for stationary sources. But the length of the normal to this surface is non-zero and therefore it is not a null surface. Particles can cross it and fly out again. The black hole itself is identified with the time-independent null surface $r = r_+ = m + (m^2 - a^2)^{1/2}$; a particle entering it cannot come out again. The stationary limit touches the black hole at the poles and bulges out at the equator. Clearly, the splitting of the two surfaces, in contrast to the Schwarzschild metric in which they were identical, is a manifestation of the rotation inherent to the space-time.

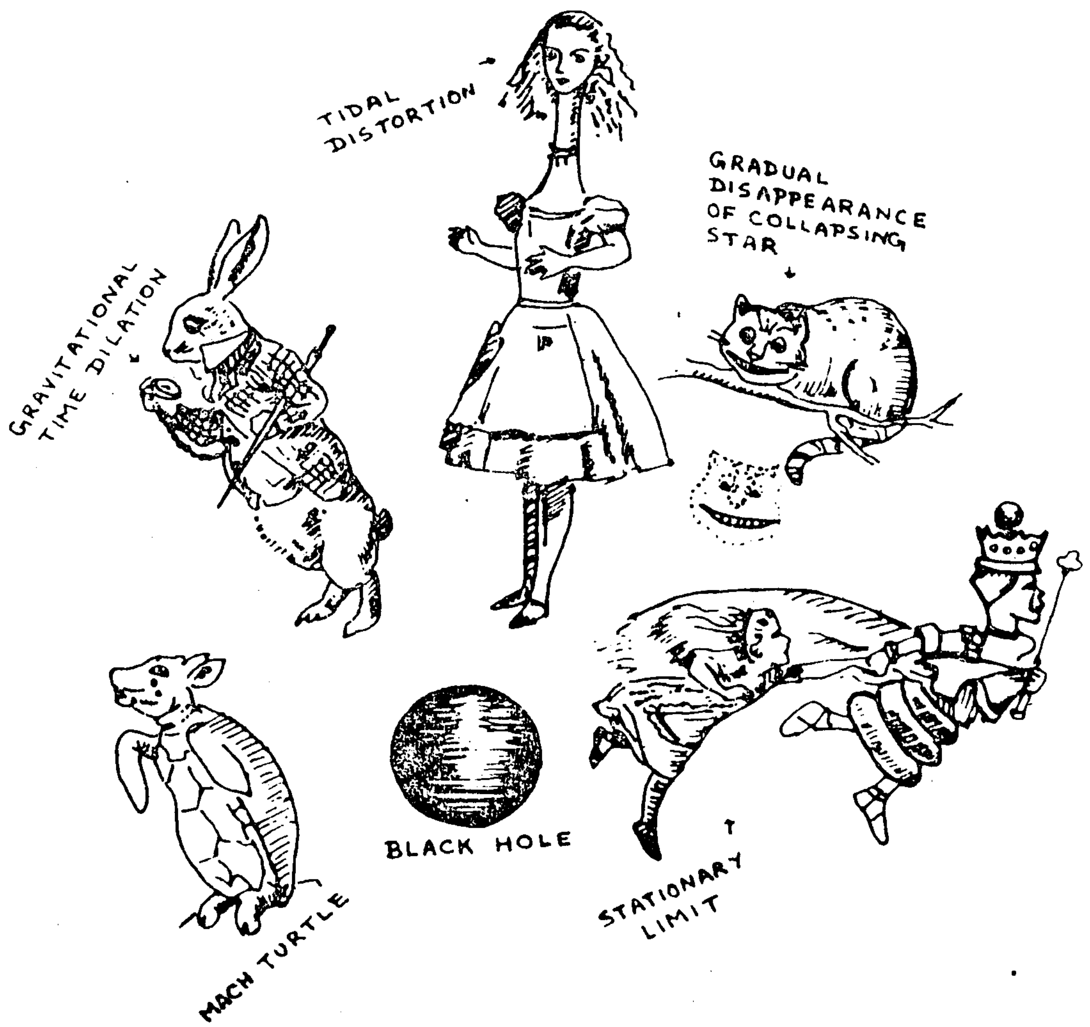
The above phenomenon can be analyzed by invoking the so-called locally non-rotating observers (or the 'rest observers') who follow the trajectories given by the vector field $\zeta^a = \left(1, 0, 0, -\frac{g_{03}}{g_{33}}\right)$, where g_{03} and g_{33} are the coefficients of the $dtd\phi$ and $d\phi^2$ terms respectively. While the Killing vector becomes null on the stationary limit, the vector ζ^a is timelike



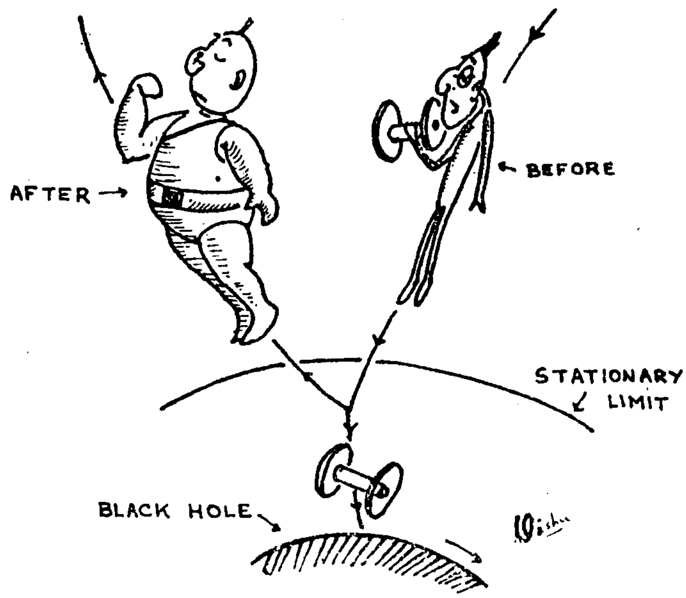
down to the black hole on which it becomes lightlike. The rotating black hole, like a whirlpool, sets up currents in the space-time that die out asymptotically at spatial infinity. Stationary observers will have to be moving against these in order to maintain constant spatial coordinates, approaching the speed of light as their location reaches the stationary limit. On the other hand the rest observers simply drift along the currents and can exist down to the black hole itself.

At this stage of exploration, the black hole physicists realized that almost all of their results had been anticipated in the latter half of the last century by an Oxford mathematics don who wrote under the name of Lewis Carroll. Alice's fall is symbolic of gravitational collapse. The white rabbit's watch cannot maintain regular time as the rabbit flits in and out of the strong gravitational field. Alice opens up like a telescope because of tidal distortion. The Cheshire cat's grin represents the gradual disappearance of a collapsing star as viewed by an outside observer. As the red queen says, an observer has to do all the running he can in order to stay at the same place as the stationary limit is reached.

The picture of the black hole that has emerged so far makes it merely the lingering ghost of a star that has long been interred in the singularity of its own making. Far away the gravitation has nothing unusual to offer; too close, the field is strong enough to pull everything in. Incapable of emitting anything and immutable, the black hole seems to have no dynamics associated with it at all. This situation changed with the coming of the Penrose process and the black hole thermo-dynamics.



The Penrose process offers, at least in principle, a method of extracting energy out of a rotating black hole. We have seen that along a geodesic with four-momentum p^a, the quantity $\xi_a p^a$ is a constant and is proportional to the energy measured by the stationary observers. It should necessarily be positive outside the stationary limit where ξ^a is timelike and stationary observers are possible; whereas this quantity, although a constant, need no longer be positive inside the stationary limit since ξ^a is now spacelike. Suppose a particle (1) falling in along a geodesic from infinity splits into two (2 and 3, both following geodesics) in the region between the stationary limit and the black hole, the so-called ergosphere. Energy conservation during this process reads, $\xi_a p_1^a = \xi_a p_2^a + \xi_a p_3^a$ more or equivalently $E_1 = E_2 + E_3$. If particle 2 escapes out, then E_2 has to be positive. On the other hand if particle 3 falls into the black hole, then E_3 could very well be negative in which case particle 2 would come out with enhanced energy ($E_2 > E_1$). At the same time the black hole's rotation slows down as a result of this energy extraction. The operation can be continued until the angular momentum of the black hole is reduced to zero. However, the geodesics involved in the process have to be stringently arranged so as to make the energy extraction feasible. Investigations have shown that there is no 'natural' way to accomplish this that would make the process astrophysically significant. Needless to say, the day will probably never arrive when superpowers will compete to cast the first stone at the black hole in order to reap the outcoming energy. Incidentally, in all our discussions, we have found that the Killing vector plays a paramount role. As the old saying goes, the spoils belong to the vector!



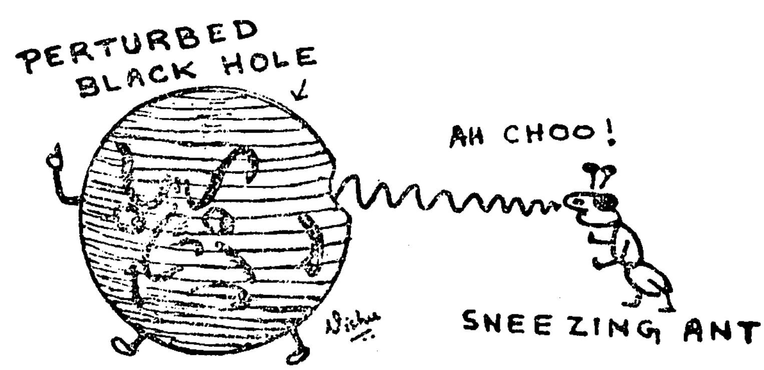
The code of conduct of black holes, stated in terms of very general laws, bears striking resemblance to ordinary thermodynamics. The analogy was initiated by the observation that the area of a black hole plays a role similar to entropy in that both of them are nondecreasing quantities in physical processes. In the simplest example of a static black hole with radially infalling particles, the area $4\pi(2m)^2$ steadily increases since particles cannot escape from within so as to decrease m.

The first law of black hole dynamics asserts that in any physical process involving black holes the energy, momentum, angular momentum and electric charge are conserved. Likewise the second law states that in any physical process the sum of the surface areas of the black holes involved can never decrease. Similarity of these two laws to the corresponding ones in thermodynamics is obvious. Better understanding of this new thermodynamics can be gained by considering the area of a Kerr black hole, $A = 4\pi(r_+^2 + a^2)$. Varying this area infinitesimally with respect to m and a and rearranging terms, it is easy to obtain the formula,

$$dm = \kappa \frac{dA}{8\pi} + \frac{a}{r_+^2 + a^2}\, da,$$

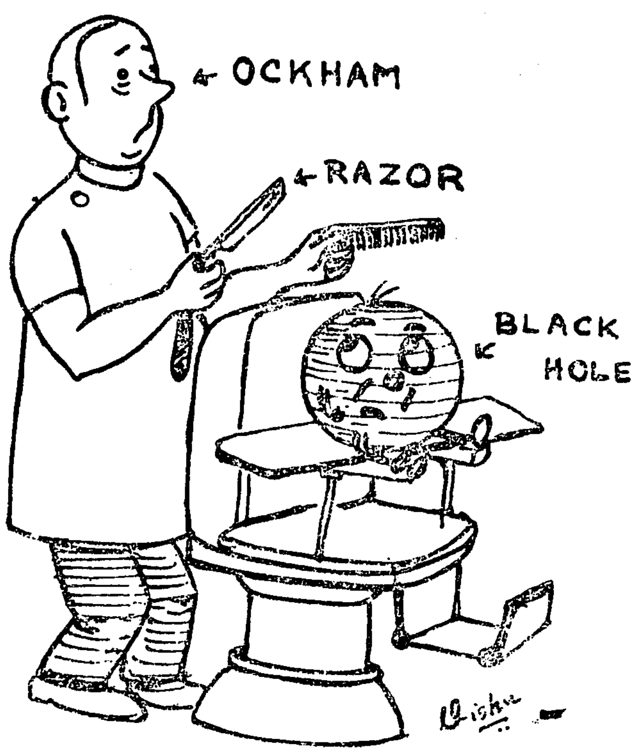
where $\kappa = (m^2 - a^2)^{1/2}/(r_+^2 + a^2)$ is the surface gravity. For a Schwarzschild black hole $\kappa = m/(2m)^2$, the acceleration due to gravity at the Schwarzschild radius. Comparing the above result with the formula for the change in energy in a thermodynamic transformation, $dE = Tds + \Omega dJ$ with Ω and J denoting angular velocity and angular momentum respectively, the correspondence between (κ, A) and (T, S) becomes evident. The analogy is further accentuated by the 'zeroth law' which showed that κ is constant over the horizon, just as a system in thermodynamic equilibrium has a constant temperature, and the third law which states that it is impossible to reduce κ to zero by any finite sequence of operations. The direct identification of κ with the temperature of the black hole results from the remarkable Hawking evaporation of a black hole in which the radiation has a black body distribution corresponding to a temperature $T = \frac{\hbar}{2\pi k}\kappa = \frac{\hbar c^3}{8\pi G k}\cdot\frac{1}{M}$, where k is the Boltzmann constant.

Long before the basic properties of black holes were elucidated, the programme of perturbing them had been launched. Side by side with the advances related to exact results, the perturbation techniques developed rapidly and were applied to a number of physical processes occurring in the gravitational fields of black holes. Either the metric is changed slightly or the curvature tensor is perturbed (Newman-Penrose formalism) and linearized Einstein equations for the perturbations are derived. Scalar, electromagnetic and Dirac (electrons and neutrinos) fields have also been studied in the Schwarzschild and the Kerr geometries. The perturbing fields are assumed to be small enough in strength so that the background geometries are considered to be unaltered. In the case of the Schwarzschild metric, the stability of the black hole against perturbations has been



rigorously proved. Scattering of waves and production of radiation by particles in motion have been studied in detail. An analysis of spherical collapse with superimposed perturbations shows that all higher order multipoles are radiated away leaving a black hole with only mass, angular momentum and charge. As for the Kerr space-time, a completely rigorous proof of stability is lacking, but both analytic and numerical calculations indicate that the black hole is in fact stable. A new feature appearing in scattering is the superradiance which is the wave analogue of the Penrose process. Waves of certain modes coming in from infinity travel out with enhanced energy extracted at the expense of the rotation of the black hole. Production of radiation from particles has also been examined.

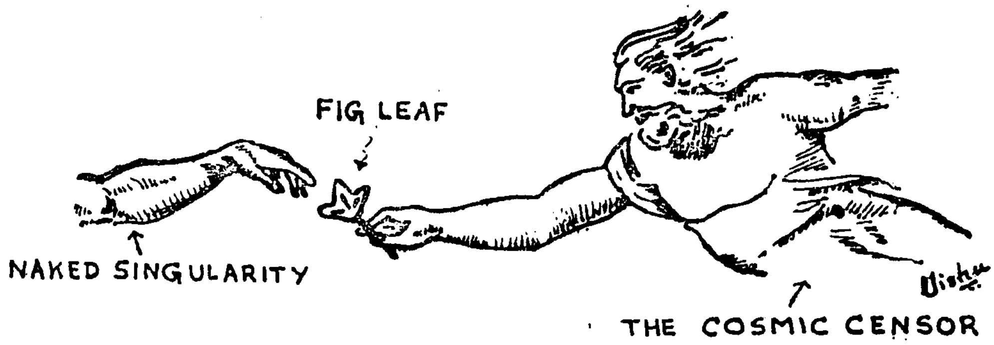
Perturbation calculations showed that a Schwarzschild black hole could not support static perturbations: they blew up either at infinity or at the horizon. In other words, it seemed that black holes distorted in shape could not exist. Similarly, as we have seen, non-spherically perturbed collapse showed that only mass, charge and angular momentum survived. These were the vague premonitions that preceded the startling Israel theorem which proved that the Schwarzschild black hole was the only static one possible in nature. Now we know that black holes come in only four varieties. By adding electric charge to the Schwarzschild and the Kerr black holes, say by rubbing them with fur, we get the Reissner-Nordstrom and the Kerr-Newman black holes respectively. Thus the black hole is characterized entirely by mass, angular momentum and charge and no other parameter. This uniqueness theorem is often referred to colloquially as the 'no-hair theorem'. The assertion that the black hole has no hair is probably the ultimate application of Ockham's razor!



How can one detect a black hole? The philosophy of detection is that of Sherlock Holmes: When you have excluded the impossible, whatever remains, however improbable, must be the truth. If the existence of an invisible object has been established by inference and if the possibility of its being a collapsed object like a black dwarf or a neutron star is ruled out, then it must be a black hole. Of course, a black hole in isolation does not betray its existence; it has to be spotted through the company it keeps. This is precisely the procedure followed in the case of Cygnus X-1, the X-ray source in Cygnus, which is strongly suspected to comprise a black hole. This is a binary star system with a visible supergiant of mass M_1 accompanied by an invisible companion of mass M_2. From the varying Doppler shift exhibited by the spectrum of the visible star, the orbital period and the velocity can at once be determined. These two quantities when fed into Kepler's law yield the mass function $f(M) = M_2^3 \sin^3 i/(M_1 + M_2)^2$, where i is the angle between the line of sight and the normal to the orbital plane which is undetermined. However, the minimum value of M_2 is obtained for $i = 90°$ with fixed M_1, which is inferred from the spectrum of the visible star. In the case of Cygnus X-1, the mass turns out to be greater than about eight times the solar mass. This is much more than the neutron star mass limit of about three solar mass. The invisible object is therefore generally believed to be a black hole. Plasma streams out from the supergiant, forms an accretion disk around the black hole and finally spirals into it. As it pours in with enormous speed, the plasma radiates by the bremsstrahlung mechanism and makes the system a source of X-rays.

Supermassive black holes of 10^6 to 10^9 solar masses have been invoked to explain the energy output of quasars. These are assumed to have resulted from the collapse of star clusters or grown out of seed black holes of about ten solar masses by accumulation of matter over a period of about a billion years. Situated in a galaxy, the black hole is fuelled by interstellar gas, stars of solar size which it can devour whole, and fragments of giants that are ripped off by tidal forces. Enormous amount of energy is liberated due to this accretion. Since quasars are observed at only cosmological distances, these cosmic fires must have been live only in the distant past. Cold remains of the once active quasars may lie buried inside many galaxies around us in the form of quiescent black holes. For all we know, there may be such a black hole some ten thousand parsecs away from us at our own galactic centre waiting to be discovered.

After a long hibernation in its mathematical lair, the black hole has travelled fast along the path of physics and has arrived in the realm of astronomy—sometimes dangerously poised on the brink of mythology. Immense amount of work has been carried out towards describing the black hole, deciphering its secrets and investigating its influence on physical processes. As Churchill might have put it, never in the field of human knowledge so much has been said by so many about something that is so close to being nothing. Still, unanswered questions and unsolved problems remain. Except under the assumption of spherical symmetry we do not have the exact scenario for gravitational collapse; we do not even possess any information on the direct formation of a rotating black hole. We do not know whether in the process of collapse a 'naked singularity', visible to outside observers, can form. Or whether Penrose's conjecture of 'cosmic censorship', which supposes that every singularity is hidden inside an event horizon, is universally valid. The singularity, naked or clothed, is the biggest problem of all. It belongs today to the



realm of conjecture and speculation, since physical laws, including general relativity, break down when confronted with the infinite space-time curvature. Will quantum effects come to the rescue, stop the collapse at the last moment and remedy the situation? No one knows. In this respect we might as well echo Faust and say, "We do not need what we know, and do not know what we need". But, one thing is certain. The black hole is by no means merely the silent finale to the symphony of the stars, nor is it the dark crystal ball in which the fate of general relativity is revealed. The unequivocal proof of black hole's existence and a proper perspective of its role in the cosmic order both lie in the domain of the future.